ST HILDA'S
ANGLICAN SCHOOL FOR GIRLS

Time:	25 minutes
Total Marks:	24 marks

Year 12 Test 2

Thursday 29th April 2021

Resource Free

ClassPad calculators are Not permitted.

Formulae Sheet is Permitted.

Name:

1. (2 , 2 = 4 marks)

Differentiate the following with respect to x. Do not simplify.

a) $3x^2e^x$

$$6xe^x + 3x^2e^x$$

b) $3e^{2x^3+1}$

$$3e^{2x^3+1} \cdot 6x^2$$

2. (2, 2 , 2, 1 = 7 marks)

-1 mark for $2 \times$ missing c

a) Evaluate the following $\int \frac{\sqrt{x}+x}{x}\,\mathrm{d}x$.

$$\int x^{-\frac{1}{2}} + 1\,dx$$

$$= 2x^{\frac{1}{2}} + x + c$$

b) Find Q in terms of p given that $\frac{dQ}{dp} = 4 - \frac{6}{p^3}$ and $Q = -3$ when $p = 1$.

$$\int 4 - 6p^{-3}dx$$

$$= 4p + 3p^{-2} + c$$

$$Q(1) = -3$$

$$\therefore c = -10$$

$$\therefore Q = 4p + \frac{3}{p^2} - 10$$

c) $\int 2x^3 e^{x^4}\,\mathrm{d}x$

$$\frac{1}{2}\int 4x^3 e^{x^4} dx$$

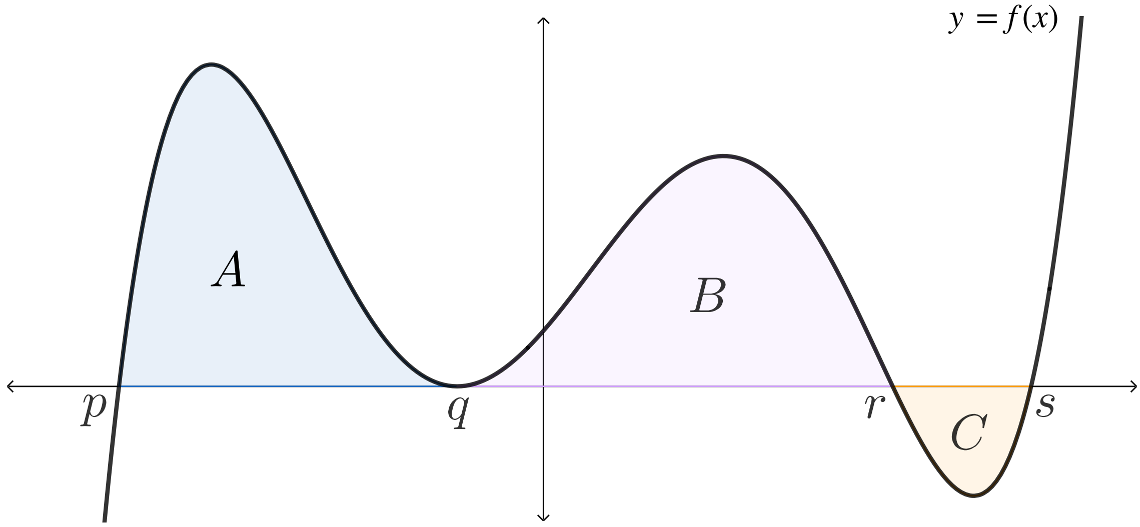
$$\frac{1}{2}e^{x^4} + c$$

d) $\frac{\mathrm{d}}{\mathrm{d}x}\int_{-2}^{x} \frac{t^2+3}{\pi - \sqrt{t}}\,\mathrm{d}t$

$$\frac{x^2+3}{\pi - \sqrt{x}}$$

3. (1, 1, 2 = 4 marks)

The three regions between the curve $y = f(x)$ and the x-axis have areas of A, B, and C units²
as shown below. Determine the following definite integrals.



a) $\int_p^s f(x)\,dx$

$A + B - C$

b) $\int_p^s 2f(x)\,dx$

$2(A + B - C)$

c) $\int_r^p f(x) + 1\,dx$

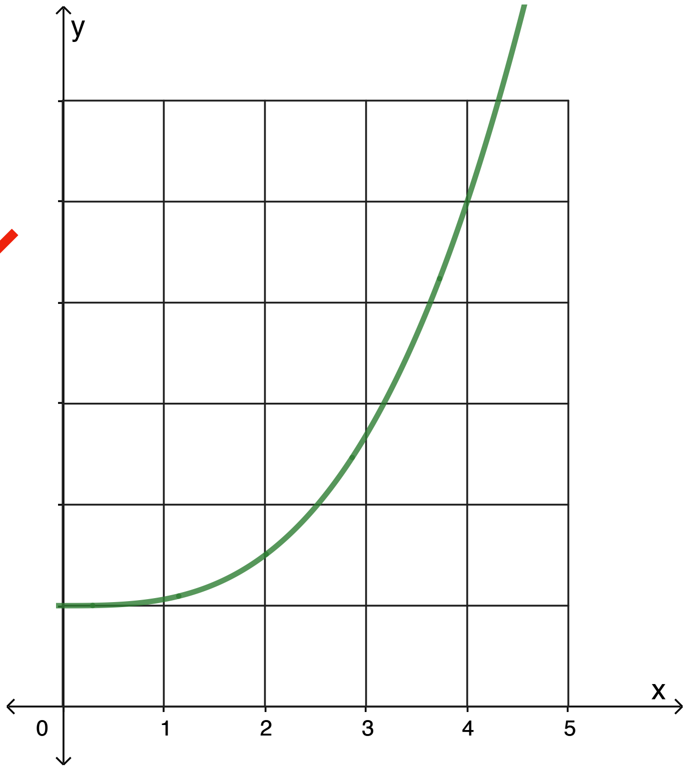
$-A - B + p - r$

4. (2 marks)

The function $f(x) = x^3 + 1$ is shown below.

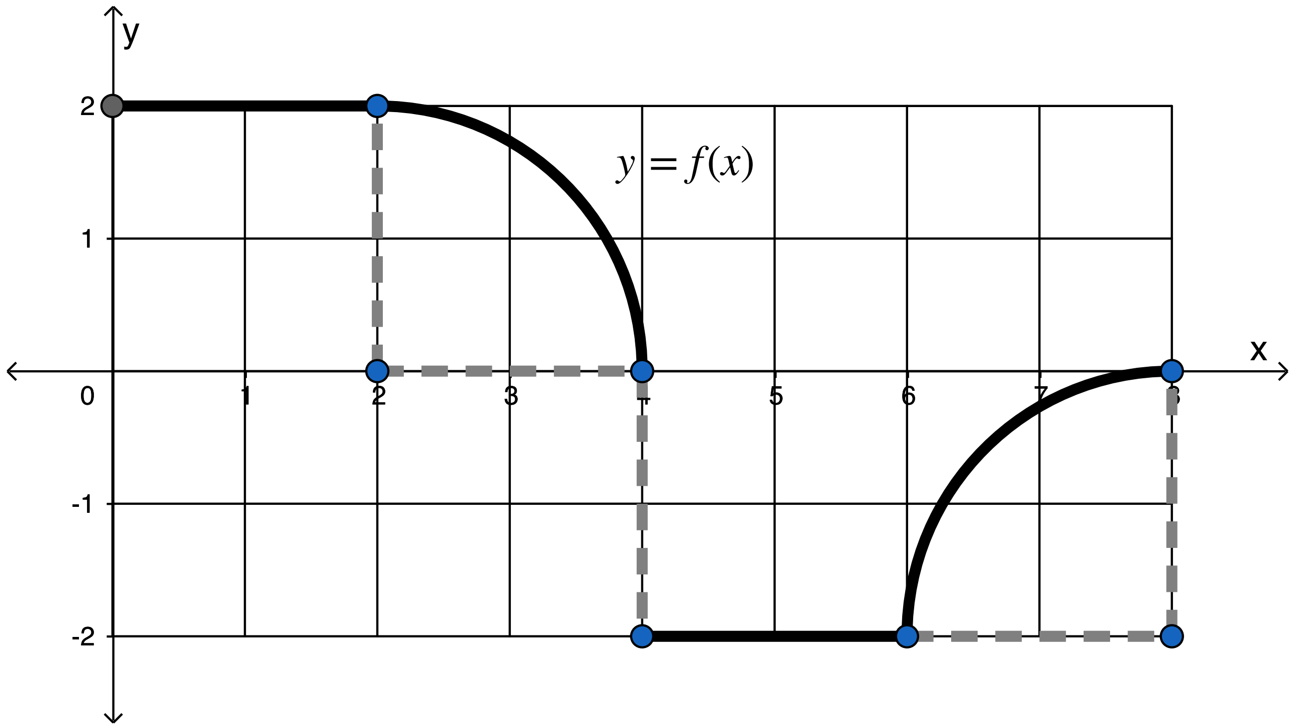
a) Using the under-estimate with widths of 1 unit, approximate the area under $f(x)$ for $1 \leq x \leq 3$. Show all working.

$$(1)\left(1^3+1\right)+(1)\left(2^3+1\right)=2+9$$
$$= 11 \text{ units}^2$$



5. (2, 2, 3 = 7 marks)

The function $f(x)$ is shown below.



a) Use the graph above to determine the following in exactly.

i. $\int_0^4 f(x)\,dx$

$\pi + 4$

ii. $\int_4^8 f(x)\,dx$

$-8 + \pi$

iii. If $\int_k^8 f(x)\,dx = 0$, solve for k.

$$2(2-k) + \pi = 8 - \pi$$

$$4 - 2k + \pi = 8 - 7)$$

$$\frac{2\pi - 4}{2} = k$$

$$k = \pi - 2$$

$k = 8$

St Hilda's
ANGLICAN SCHOOL FOR GIRLS

Time:	20 minutes
Total Marks:	20 marks

Year 12 Test 2

Thursday 29th April 2021

Resource Assumed

ClassPad calculators are permitted.
Formulae Sheet is Permitted.

Name:

6. (2, 2 = 4 marks)

An imaginary radioactive isotope Coraronium decays at a rate of $\frac{\mathrm{d}A}{\mathrm{d}t} = -0.14A$ where A (kg) is the amount of Coraronium remaining and t is in years.

a) If 2 kg of Coraronium exists originally, determine how much will remain after 10 years.

$$A = 2e^{-0.14t}$$
$$A(10) = 0.49\ kg$$

b) Determine the half life of Coraronium, that is the time it takes for the radioactive isotope to be reduced to 50%.

$$\frac{1}{2} = e^{-0.14t}$$
$$t = 4.95 \text{ years}$$

7. (1, 2 = 3 marks)

A population changes such that $\dfrac{\mathrm{d}P}{\mathrm{d}t} = -0.12P$, where t is in years.

a) Is the the population growing or decaying?

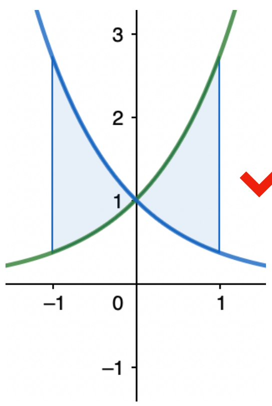
Decaying

b) If the population is 120 000 after 8 years. Calculate (to the nearest 1000) the original population.

$$P = P_0 \cdot e^{-0.12t}$$
$$120000 = P_0 \cdot e^{-0.12(8)}$$
$$P_0 = 313000$$

8. (4 marks)

Given $f(x) = \mathrm{e}^x$ and $g(x) = \mathrm{e}^{-x}$ find the **exact** area of the regions enclosed by the two functions, $x = -1$ and $x = 1$. Show the use of a sketch in your solution.



$$\text{Area } = 2\int_0^1 e^x - e^{-x} dx$$
$$= 2\left[e^x + e^{-x}\right]_0^1$$
$$= 2\left[\left(e^1 + e^{-1}\right) - (1+1)\right]$$
$$= 2\left(e^1 - \frac{1}{e} - 2\right)$$
$$= 2e + 2e^{-1} - 4$$

9. (3, 2, 3, 1 = 9 marks)

A particle's is moving with rectilinear motion and its position can be modelled by the function $v(t) = 3t^2 - 12t + 9$ for $0 \leq t \leq 4$, where v is measured in metres/seconds and t is measured in seconds.

a) Determine when the velocity of the particle is maximised.

$v(t) = 3(t-3)(t-1)$ ✓

Reasoning

Working ✓

$t = 0, t = 4$ ✓

b) If the particle is initially at the origin determine an expression for the displacement.

$$x(t) = t^3 - 6t^2 + 9t + c$$
$$x(0) = 0 \quad \therefore c = 0$$ ✓
$$\therefore x(t) = t^3 - 6t^2 + 9t$$ ✓

c) Determine the total distance travelled in the first 3 seconds.

$$v(t) = 0 \quad @ \quad t = 1 \quad \& \quad t = 3$$ ✓
$$x(0) = 0$$
$$x(1) = 4$$ ✓
$$x(3) = 0$$
$\therefore$ dist $= 8$ m ✓

d) Determine the change in displacement in the 2nd second.

-2 units ✓